Comment on “$1/f$ noise in the Bak-Sneppen model”

Jörn Davidsen* and Norbert Lüthje†
Institut für Theoretische Physik und Astrophysik, Christian-Albrechts-Universität, Olshausenstraße 40, 24118 Kiel, Germany

(August 18, 2000)

Contrary to the recently published results by Daerden and Vanderzande (Phys. Rev. E **53**, 4723 (1996)), we show that the time correlation function in the random neighbor version of the Bak-Sneppen model can be well approximated by an exponential giving rise to a $1/f^2$ power spectrum.

5.40-a, 64.60.Ak, 87.10+e

Recently, an exact solution of the random neighbor version of the Bak-Sneppen model was presented by de Boer and co-workers [1]. They derived a master equation for the probability $P_n(t)$ that n of out of N numbers have a value less than a fixed value λ at (discrete) time t. In the limit $N \to \infty$ and $\lambda = 1/2$, P_n has the scaling form $P_n(t) = \frac{1}{\sqrt{N}} f(x = n/\sqrt{N}, \tau = t/N)$. Inserting this expression into the master equation gives the following Fokker-Planck equation for $f(x,\tau)$ with a reflecting boundary at $x = 0$:

$$\frac{\partial f}{\partial \tau} = \frac{1}{4}\,\frac{\partial^2 f}{\partial x^2} + \frac{\partial}{\partial x}(xf). \tag{1}$$

Consequently, the random neighbor version of the Bak-Sneppen model for $N \to \infty$ is just an Ornstein-Uhlenbeck process, i.e., Brownian motion in a parabolic potential. Given the initial condition $f(x,0) = \delta(x-y)$, the solution is

$$f(x,\tau) = \sqrt{\frac{2}{\pi(1-\exp^{-2\tau})}}\,\exp^{\frac{-2(x-y\exp^{-\tau})^2}{1-\exp^{-2\tau}}}. \tag{2}$$

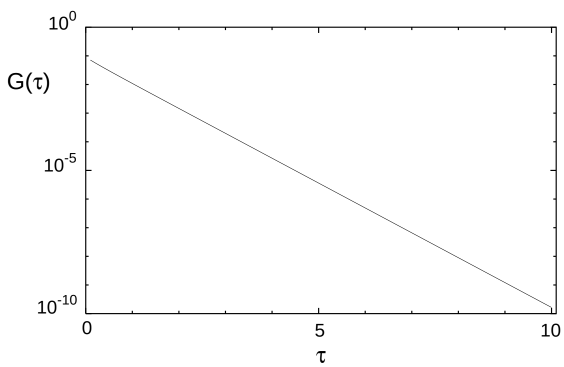

FIG. 1. Plot of the correlation function given by Eq. 6.

It follows for the autocorrelation function $G(\tau)$ of the time signal $x(\tau)$

$$G(\tau) = \frac{1}{4}\exp^{-a|\tau|}, \tag{3}$$

where $a \equiv 1$. This directly gives the power spectral density $S(\tilde{\omega})$ via a Fourier transform of $G(\tau)$

$$S(\tilde{\omega}) = \frac{1}{2}\frac{a}{a^2 + \tilde{\omega}^2}. \tag{4}$$

Going back to the unscaled variables leads to

$$S_P(\omega) = \frac{1}{2}\frac{a}{\frac{a^2}{N^2} + \omega^2}, \tag{5}$$

which is only valid for low frequencies. Hence, the power spectral density of the signal $n(t)$ decays as $1/f^2$ and for very low frequencies it even becomes constant. However, the above calculation was carried out without applying the boundary condition at $x = 0$. Nevertheless, it is already clear from a physical point of view that the functional form of $G(\tau)$ will not change drasticly by incorporating a reflecting boundary. This is supported mathematically by the fact that one simply has to use the method of images. This was done in Ref. [2] giving

$$\begin{aligned} G(\tau) = \frac{1}{8\pi}&[1-\exp^{-2\tau}]^{3/2}[F(1,2,3/2,r_-(\tau)) \\ &+F(1,2,3/2,r_+(\tau)) - F(1,2,5/2,r_-(\tau))/3 \\ &-F(1,2,5/2,r_+(\tau))/3] - \frac{1}{2\pi}, \end{aligned} \tag{6}$$

where $F(a,b,c,z)$ is the hypergeometric function and where $r_\pm(\tau) = \frac{1}{2}[1 \pm \exp(-\tau)]$. In Fig. 1, $G(\tau)$ is shown. We clearly find an exponential behavior with $a = 0.869 \pm 0.008$ for $0.1 < \tau < 10$ giving rise to a power spectral density as in Eq. (4). This is confirmed by a numerical Fourier transform of Eq. (6) (see Fig. 2). Here, it has to be noted that $G(\tau)$ is an even function, i.e., $G(\tau) = G(-\tau)$. This also ensures that the Fourier transform $S(\tilde{\omega})$ is real.

In the case of the Bak-Sneppen model with one next neighbor, we also cannot confirm the results presented in Ref. [2]. A direct simulation of the time signal gives $S_P(\omega) \propto 1/\omega^{1.5}$ over 2 decades for a system size of $N = 8192$. Hence, although the power spectral density in the Bak-Sneppen model decays as a power law, the exponent is far from one. This is also true for a different definition of the time signal [3].

In conclusion, there is no sign of $1/f$ noise in the random neighbor version of the Bak-Sneppen model and even in the next neighbor version there is no $1/f$ noise in the strict sense.

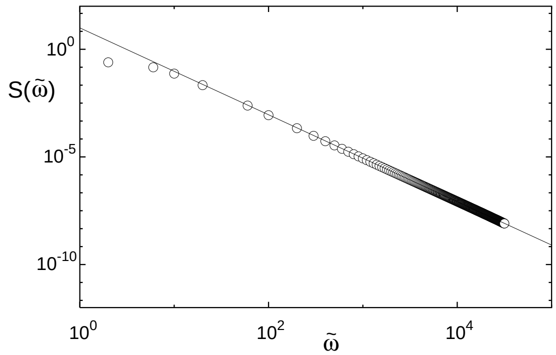

FIG. 2. Plot of the power spectrum given by a numerical Fourier transformation of Eq. 6. The solid line with exponent -2 is drawn for reference.

ACKNOWLEDGMENTS

J. Davidsen would like to thank the Land Schleswig-Holstein, Germany, for the financial support granted to him.

---

* Email address: davidsen@theo-physik.uni-kiel.de

† Email address: luethje@theo-physik.uni-kiel.de

[1] J. de Boer *et. al.*, Phys. Rev. Lett. **73**, 906 (1994).

[2] F. Daerden and C. Vanderzande, Phys. Rev. E **53**, 4723 (1996).

[3] M. Paczuski, S. Maslov, and P. Bak, Phys. Rev. E **53**, 4723 (1996); S. Maslov *et. al.*, Phys. Rev. E **58**, 7141 (1998).